Задача 1.2. (44)

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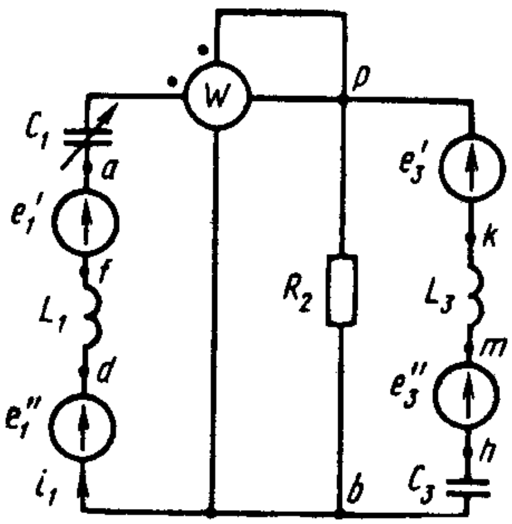

Рис. 1.29.

$L_1 = 0.68$ мГн;

$L_3 = 2.73$ мГн;

$C_1 = 1.62$ мкФ;

$C_3 = \infty$ мкФ;

$R_2 = 65$ Ом;

$f = 1400$ Гц;

$e'_1 = 141 \cdot sin\omega t$, В;

$e''_1 = 0$;

$e'_3 = 181.4 \cdot sin\omega t$, В;

$e''_3 = 216 \cdot cos(\omega t - 180°) = 216 \cdot sin(\omega t - 90°)$, В.

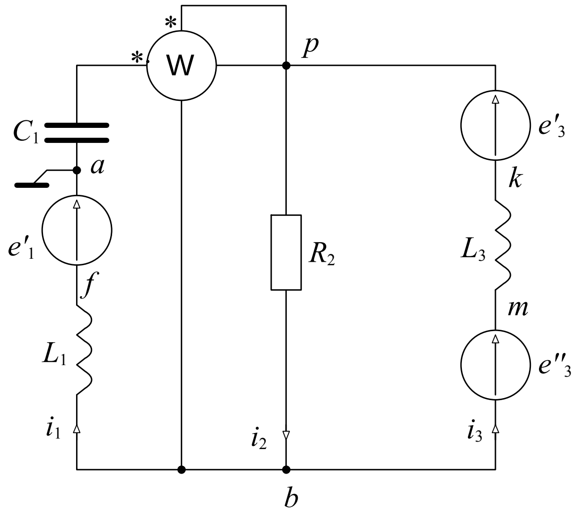

Рис. 1.

1. Система уравнений для расчета токов:

- в дифференциальной форме

$$i_1 + i_3 - i_2 = 0$$

$$L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1 dt + i_2 R_2 = e_1' = e_1$$

$$i_2 R_2 + L_3 \frac{di_3}{dt} = e_3' + e_3'' = e_3$$

- в символической форме

$$\underline{I}_1 + \underline{I}_3 - \underline{I}_2 = 0$$

$$j\omega L_1 \cdot \underline{I}_1 + \frac{1}{j\omega C_1} \cdot \underline{I}_1 + R_2 \cdot \underline{I}_2 = \underline{E}'_1 = \underline{E}_1$$

$$R_2 \cdot \underline{I}_2 + j\omega L_3 \cdot \underline{I}_3 = \underline{E}'_3 + \underline{E}''_3 = \underline{E}_3$$

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2. Расчет комплексов действующих значений ЭДС и токов в ветвях (рис. 1).

$e'_1 = 141 \cdot sin\omega t$, B; $\quad$ => $\underline{E}'_1 = \frac{141}{\sqrt{2}} \cdot e^{j\,0\,^\circ}$, B;

$e'_3 = 181.4 \cdot sin\omega t$, B; $\quad$ => $\underline{E}'_3 = \frac{181.4}{\sqrt{2}} \cdot e^{j\,0\,^\circ}$, B;

$e''_3 = 216 \cdot sin(\omega t - 90^\circ)$, B; $\quad$ => $\underline{E}''_3 = \frac{216}{\sqrt{2}} \cdot e^{-j\,90\,^\circ}$, B;

$\underline{E}_1 = \underline{E}'_1 = 100 \cdot e^{j\,0\,^\circ}$, B;

$\underline{E}_3 = \underline{E}'_3 + \underline{E}''_3 = 128.27 \cdot e^{j\,0\,^\circ} + 152.74 \cdot e^{-j\,90\,^\circ} = 199 \cdot e^{-j\,50\,^\circ}$, B;

$\omega = 2 \cdot \pi \cdot f = 2 \cdot \pi \cdot 1400 = 8.7965 \times 10^3$ 1/c;

$X_{L1} = \omega \cdot L_1 = 8.7965 \cdot 10^3 \cdot 6.8 \times 10^{-4} = 5.982$ Ом;

$X_{C1} = \frac{1}{\omega \cdot C_1} = \frac{1}{8.7965 \times 10^3 \cdot 1.62 \times 10^{-6}} = 70.174$ Ом;

$X_{L3} = \omega \cdot L_3 = 8.7965 \times 10^3 \cdot 2.73 \times 10^{-3} = 24.014$ Ом;

$\underline{Z}_1 = jX_{L1} - jX_{C1} = j\,(5.982 - 70.174) = -64.192j = 64.2 \cdot e^{-j\,90\,^\circ}$ Ом;

$\underline{Z}_2 = R_2 = 65$ Ом;

$\underline{Z}_3 = jX_{L3} = j\,20.02 = 24 \cdot e^{j\,90\,^\circ}$ Ом.

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Исходную схему замещения (рис. 1) представим в следующем виде (рис. 2).

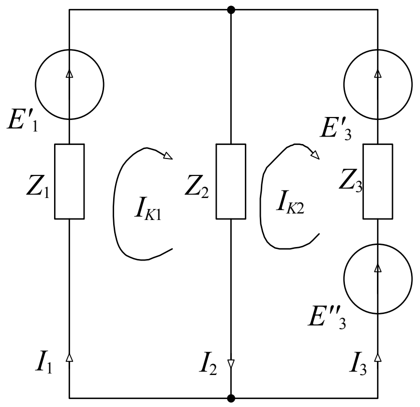

Рис. 2.

В схеме (рис. 2) комплексы действующих значений токов $\underline{I}_1$, $\underline{I}_2$ и $\underline{I}_3$ рассчитаем методом контурных токов.

$$\begin{cases}(\underline{Z}_1+\underline{Z}_2)\cdot\underline{I}_{K1}-\underline{Z}_2\cdot\underline{I}_{K2}=\underline{E}_1\\ -\underline{Z}_2\cdot\underline{I}_{K1}+(\underline{Z}_2+\underline{Z}_3)\cdot\underline{I}_{K2}=-\underline{E}_3\end{cases}$$

$$\underline{Z}_1+\underline{Z}_2=-64.2j+65=65-64.2j=91.35\cdot e^{-j\,44.64\,^\circ}\ \text{Ом};$$

$$\underline{Z}_2+\underline{Z}_3=65+24j=69.29\cdot e^{j\,20.28\,^\circ}\ \text{Ом};$$

$$-\underline{Z}_2=-65=65\cdot e^{j\,180\,^\circ}\ \text{Ом};$$

$$\underline{E}_1=100\ \text{В};$$

$$-\underline{E}_3=-199\cdot e^{-j\,50\,^\circ}=199\cdot e^{j\,130\,^\circ}\ \text{В};$$

$$\begin{cases}91.35\cdot e^{-j\,44.64\,^\circ}\cdot\underline{I}_{K1}+65\cdot e^{j\,180\,^\circ}\cdot\underline{I}_{K2}=100\\ 65\cdot e^{j\,180\,^\circ}\cdot\underline{I}_{K1}+69.29\cdot e^{j\,20.28\,^\circ}\cdot\underline{I}_{K2}=199\cdot e^{j\,130\,^\circ}\end{cases}$$

$$\underline{\Delta}=\left|\begin{pmatrix}91.35\cdot e^{-j\,44.64\,^\circ} & 65\cdot e^{j\,180\,^\circ}\\ 65\cdot e^{j\,180\,^\circ} & 69.29\cdot e^{j\,20.28\,^\circ}\end{pmatrix}\right|=$$

$$=91.35\cdot e^{-j\cdot 44.64\cdot\,^\circ}\cdot 69.29\cdot e^{j\,20.28\,^\circ}-\left(65\cdot e^{j\,180\,^\circ}\right)^2=1.54\times10^3-2.61j\times10^3=3.03\times10^3\cdot e^{-j\,59.45\,^\circ};$$

$$\underline{\Delta}_1=\left|\begin{pmatrix}100 & 65\cdot e^{j\,180\,^\circ}\\ 199\cdot e^{j\,130\,^\circ} & 69.29\cdot e^{j\,20.28\,^\circ}\end{pmatrix}\right|=$$

$$=100\cdot 69.29\cdot e^{j\cdot 20.28\cdot\,^\circ}-199\cdot e^{j\cdot 130\cdot\,^\circ}\cdot 65\cdot e^{j\,180\,^\circ}=-1.81\times10^3+1.23j\times10^4=1.25\times10^4\cdot e^{j\,98.57\,^\circ};$$

$$\underline{\Delta}_2=\left|\begin{pmatrix}91.35\cdot e^{-j\,44.64\,^\circ} & 100\\ 65\cdot e^{j\,180\,^\circ} & 199\cdot e^{j\,130\,^\circ}\end{pmatrix}\right|=$$

$$=91.35\cdot e^{-j\cdot 44.64\cdot\,^\circ}\cdot 199\cdot e^{j\cdot 130\cdot\,^\circ}-65\cdot e^{j\cdot 180\cdot\,^\circ}\cdot 100=7.97\times10^3+1.81j\times10^4=1.98\times10^4\cdot e^{j\,66.37\,^\circ};$$

$$\underline{I}_{K1} = \frac{1.25\times10^4\cdot e^{j\,98.57\,^\circ}}{3.03\times10^3\cdot e^{-j\,59.45\,^\circ}} = 4.11\cdot e^{j\,158.02\,^\circ} = -3.81 + 1.54j\ \text{ A};$$

$$\underline{I}_{K2} = \frac{1.98\times10^4\cdot e^{j\,66.37\,^\circ}}{3.03\times10^3\cdot e^{-j\,59.45\,^\circ}} = 6.54\cdot e^{j\,125.81\,^\circ} = -3.83 + 5.3j\ \text{ A};$$

$$\underline{I}_1 = \underline{I}_{K1} = 4.11\cdot e^{j\,158.02\,^\circ} = -3.81 + 1.54j\ \text{ A};$$

$$\underline{I}_2 = \underline{I}_{K1} - \underline{I}_{K2} = -3.81 + 1.54j - (-3.83 + 5.3j) = 0.02 - 3.76j = 3.76\cdot e^{-j\,89.77\,^\circ}\ \text{A};$$

$$\underline{I}_3 = -\underline{I}_{K2} = -\left(6.54\cdot e^{j\,125.81\,^\circ}\right) = 6.54\cdot e^{-j\,54.19\,^\circ}\ \text{A}.$$

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Баланс мощностей

$$\underline{S}_{ucm} = \underline{E}_1\cdot\underline{I}_1^* + \underline{E}_3\cdot\underline{I}_3^* =$$

$$= 100\cdot e^{j\,0\,^\circ}\cdot4.11\cdot e^{-j\,158.02\,^\circ} + 199.5\cdot e^{-j\,50\,^\circ}\cdot6.54\cdot e^{j\,54.19\,^\circ} = 920.1 - 58.5j\ \text{ ВА};$$

$$P_{ucm} = 920.1\ \text{Вт};\quad Q_{ucm} = 58.5\ \text{вар (емк)}.$$

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$$P_{np} = I_2^2\cdot R_2 = 3.76^2\cdot65 = 918.9\ \text{ Вт};$$

$$Q_{np} = I_1^2\cdot(X_{L1} - X_{C1}) + I_3^2\cdot X_{L3} = 4.11^2\cdot(5.98 - 70.17) + 6.54^2\cdot24.01 = -57.4\ \text{ вар};$$

Погрешность расчета

$$\delta_{P\%} = \frac{|P_{ucm} - P_{np}|}{P_{ucm}}\cdot100\% = \frac{|920.1 - 918.9|}{920.1}\cdot100 = 0.1\ \%$$

$$\delta_{Q\%} = \frac{|Q_{ucm} - Q_{np}|}{Q_{ucm}}\cdot100\% = \frac{|58.5 - 57.4|}{58.5}\cdot100 = 1.9\ \%$$

3. Показание ваттметра.

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$$\underline{U}_W = \underline{U}_{pb} = \underline{I}_2\cdot\underline{Z}_2 = 3.76\cdot e^{-j\,89.77\,^\circ}\cdot65 = 244.6\cdot e^{-j\,89.77\,^\circ}\ \text{В};$$

$$\underline{I}_W = \underline{I}_1 = 4.11\cdot e^{j\cdot158.02\cdot\,^\circ}\ \text{А};$$

$$P_W = \text{Re}(\underline{U}_W\cdot\underline{I}_W^*) = Re\left(244.6\cdot e^{-j\,89.77\,^\circ}\cdot4.11\cdot e^{-j\,158.02\,^\circ}\right) = -380\ \text{ Вт}.$$

Расчет потенциалов отдельных точек схемы.

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$\underline{\phi}_a = 0;$

$\underline{\phi}_p = \underline{\phi}_a - \underline{I}_1 \cdot (-jX_{C1}) = 0 - (-3.81 + 1.54j) \cdot (-j \cdot 70.17) = -108.1 - 267.3j$ В;

$\underline{\phi}_f = \underline{\phi}_a - \underline{E}'_1 = 0 - 100 = -100$ В;

$\underline{\phi}_b = \underline{\phi}_f + \underline{I}_1 \cdot (jX_{L1}) = -100 + (-3.81 + 1.54j) \cdot j \cdot 5.98 = -109.2 - 22.8j$ В;

$\underline{\phi}_m = \underline{\phi}_b + \underline{E}''_3 = (-109.2 - 22.8j) + (-152.7j) = -109.2 - 175.5j$ В;

$\underline{\phi}_k = \underline{\phi}_p - \underline{E}'_3 = (-108.1 - 267.3j) - 128.3 = -236.4 - 267.3j$ В.

Векторная лучевая диаграмма токов и векторная топографическая диаграмма напряжений приведены на рис. 3.

5. Мгновенное значение тока $i_{КЗ}$ (ток короткого замыкания емкости C_1).
   Для нахождения $i_{КЗ}$ воспользуемся МЭГ (рис. 4).

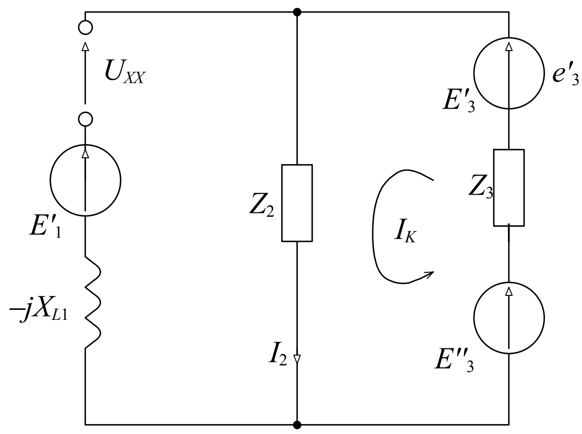

Рис. 4.

$$\underline{I}_2 = \underline{I}_K = \frac{\underline{E}_3}{\underline{Z}_2 + \underline{Z}_3}; \quad \underline{U}_{XX} + \underline{I}_2 \underline{Z}_2 = \underline{E}_1;$$

$$\underline{E}_{ЭГ} = \underline{U}_{XX} = \underline{E}_1 - \underline{I}_2 \underline{Z}_2;$$

$$\underline{Z}_{ЭГ} = -jX_{L1} + \frac{\underline{Z}_2 \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3};$$

$$\underline{I}_{КЗ} = \frac{\underline{E}_{ЭГ}}{\underline{Z}_{ЭГ}}.$$

Находим

$$\underline{I}_2 = \frac{\underline{E}_3}{\underline{Z}_2 + \underline{Z}_3} = \frac{199 \cdot e^{-j \cdot 50 \cdot °}}{69.29 \cdot e^{j 20.28°}} = 0.969 - 2.704j \text{ А};$$

$$\underline{E}_{ЭГ} = \underline{E}_1 - \underline{I}_2 \underline{Z}_2 = 100 - (0.973 - 2.709j) \cdot 65 = 36.8 + 176.1j = 180 \cdot e^{j\,78.29\,°} \text{ В};$$

$$\underline{Z}_{ЭГ} = -jX_{L1} + \frac{\underline{Z}_2 \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = -j \cdot 5.982 + \frac{65 \cdot 24j}{69.29 \cdot e^{j\,20.28\,°}} = 7.8 + 15.14j = 17 \cdot e^{j\,62.74\,°} \text{ Ом};$$

$$\underline{I}_{КЗ} = \frac{\underline{E}_{ЭГ}}{\underline{Z}_{ЭГ}} = \frac{180 \cdot e^{j\,78.29\,°}}{17 \cdot e^{j\,62.74\,°}} = 10 \cdot e^{j\,15.6\,°} \text{ А};$$

$$\underline{I}_{КЗ} = 10 \cdot e^{j\,15.6\,°} \text{ А} \Rightarrow i_{КЗ}(\omega t) = 10 \cdot \sqrt{2} \cdot \sin(\omega t + 15.6°), \text{ А}.$$

График изменения тока $i_{КЗ}(\omega t)$ построен на рис. 5.

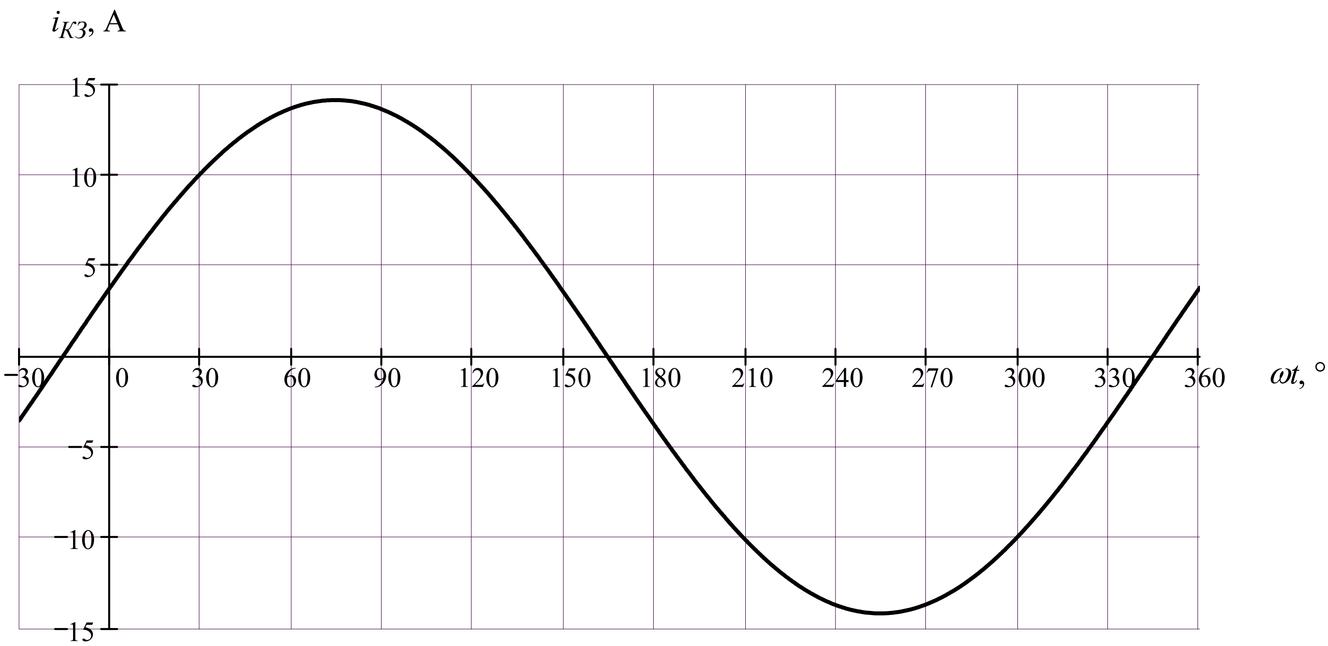

Рис. 5.

6. Система уравнений для расчета токов в цепи с взаимной индуктивной связью (рис. 6).

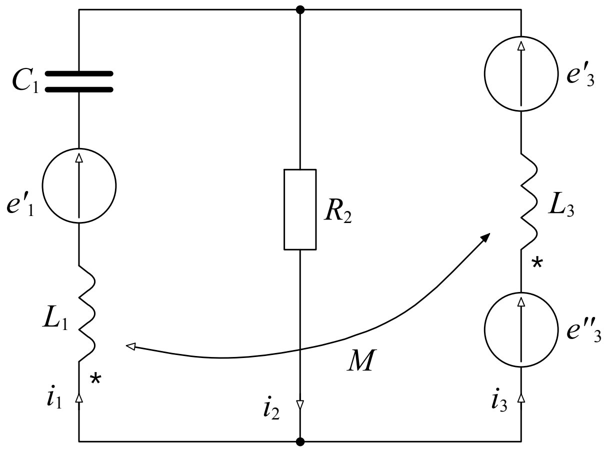

Рис. 6.

- в дифференциальной форме

$$i_1 + i_3 - i_2 = 0$$

$$L_1 \frac{di_1}{dt} + M \frac{di_3}{dt} + \frac{1}{C_1}\int i_1 dt + i_2 R_2 = e_1' = e_1$$

$$i_2 R_2 + L_3 \frac{di_3}{dt} + M \frac{di_1}{dt} = e_3' + e_3'' = e_3$$

- в символической форме

$$\underline{I}_1 + \underline{I}_3 - \underline{I}_2 = 0$$

$$j\omega L_1 \cdot \underline{I}_1 + j\omega M \cdot \underline{I}_3 + \frac{1}{j\omega C_1} \cdot \underline{I}_1 + R_2 \cdot \underline{I}_2 = \underline{E}'_1$$

$$R_2 \cdot \underline{I}_2 + j\omega L_3 \cdot \underline{I}_3 + j\omega M \cdot \underline{I}_1 = \underline{E}'_3 + \underline{E}''_3$$

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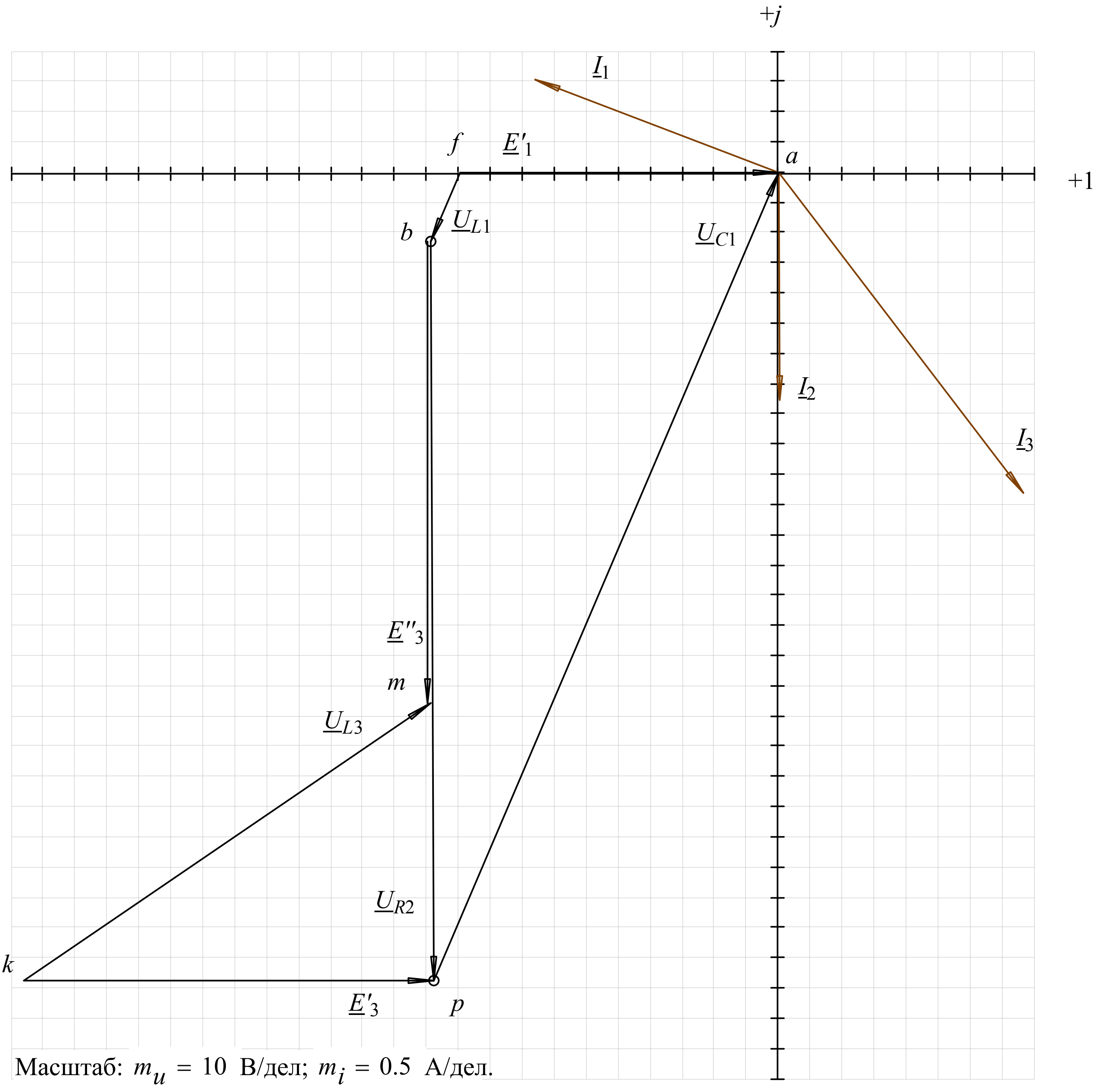

+j
+1
I1
E'1
f
a
b
UL1
UC1
I2
I3
E''3
m
UL3
UR2
k
p
E'3
Масштаб: mu = 10 В/дел; mi = 0.5 А/дел.

Рис. 3